



REAL-TIME ALGORITHMS FOR DIGITAL TWINS

Introduction to Digital Twins

UKACM Autumn School 2026
Prof. Dirk Hartmann

Real-Time Algorithms for Digital Twins

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Real-Time Algorithms for Digital Twins

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Algorithms for Real-time Digital Twins

A curated collection of compact, educational algorithms in the [Julia Programming Language](#) illustrating core numerical methods for real-time digital twins. The collection forms the core of my lecture on *Algorithms for Real-time Digital Twins*. It focuses on dynamical systems, including ordinary and partial differential equations, as well as model order reduction and data-driven discovery modeling. The algorithms are demonstrated through a set of simplified examples. All examples are self-contained, minimal, and designed for classroom use. The emphasis is on simplicity rather than completeness or effectiveness; for example, consistently simple first- or second-order time-stepping schemes are used.

The collection is inspired by the famous *99-line* [2] and *88-line* [3] topology optimization papers, as well as the *Numerical Recipes* series [1]. Many examples are also inspired by [4, 5].

PCB transient thermal simulation

Lorenz system trajectory

About

A curated collection of compact, educational algorithms in Julia illustrating core numerical algorithms for real-time digital twins. The collection is inspired by the famous "99-line" and "88-line" topology optimization papers as well as the "Numerical Recipes" series. All examples are self-contained, minimal, and designed for classroom use.

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Contributors 1

Dirk-Hartmann Dirk Hartmann

Languages

Julia 100%

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<https://github.com/Dirk-Hartmann/real-time-digital-twins>



1

Digital Engineering

2

Digital Twins

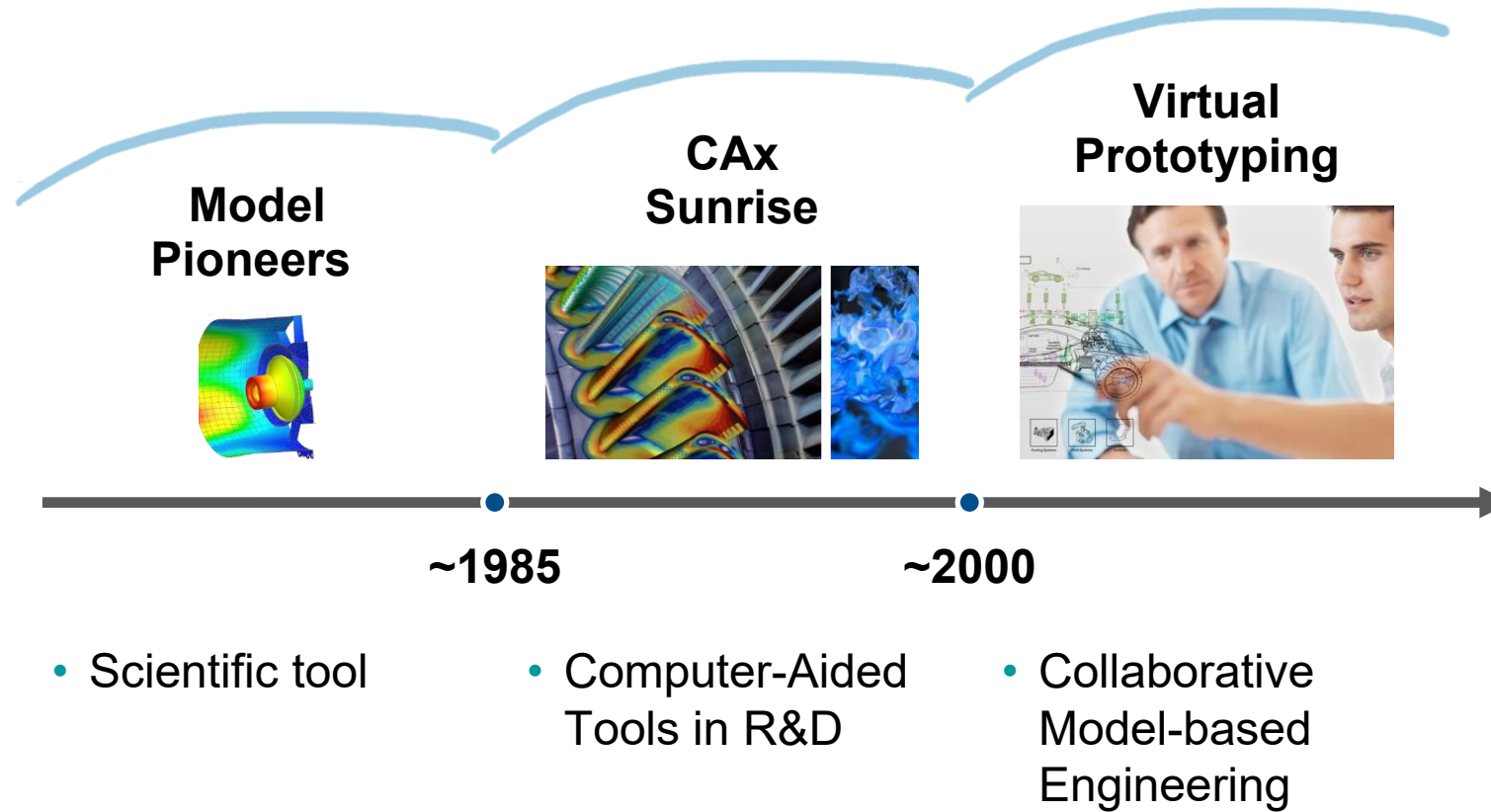
3

Real-time Algorithms

4

Examples

Digital Twin – *An old story!*



From human computers to autonomous computational engineers



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Anthony Greicius (2016): [When Computers Were Human](#). NASA Blog

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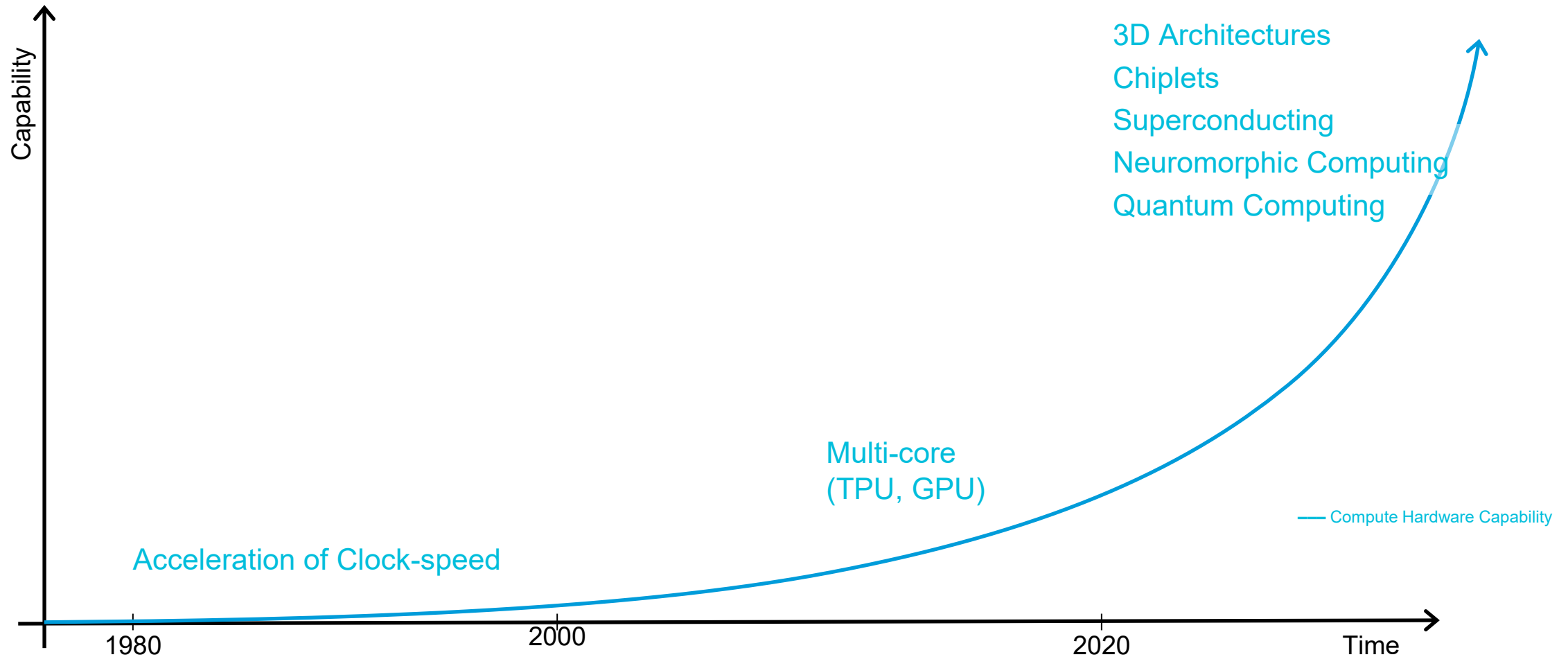
Real-Time Algorithms for Digital Twins | Prof. Dirk Hartmann

From human computers to autonomous computational engineers

Electronics is evolving delivering more than Moore's law.



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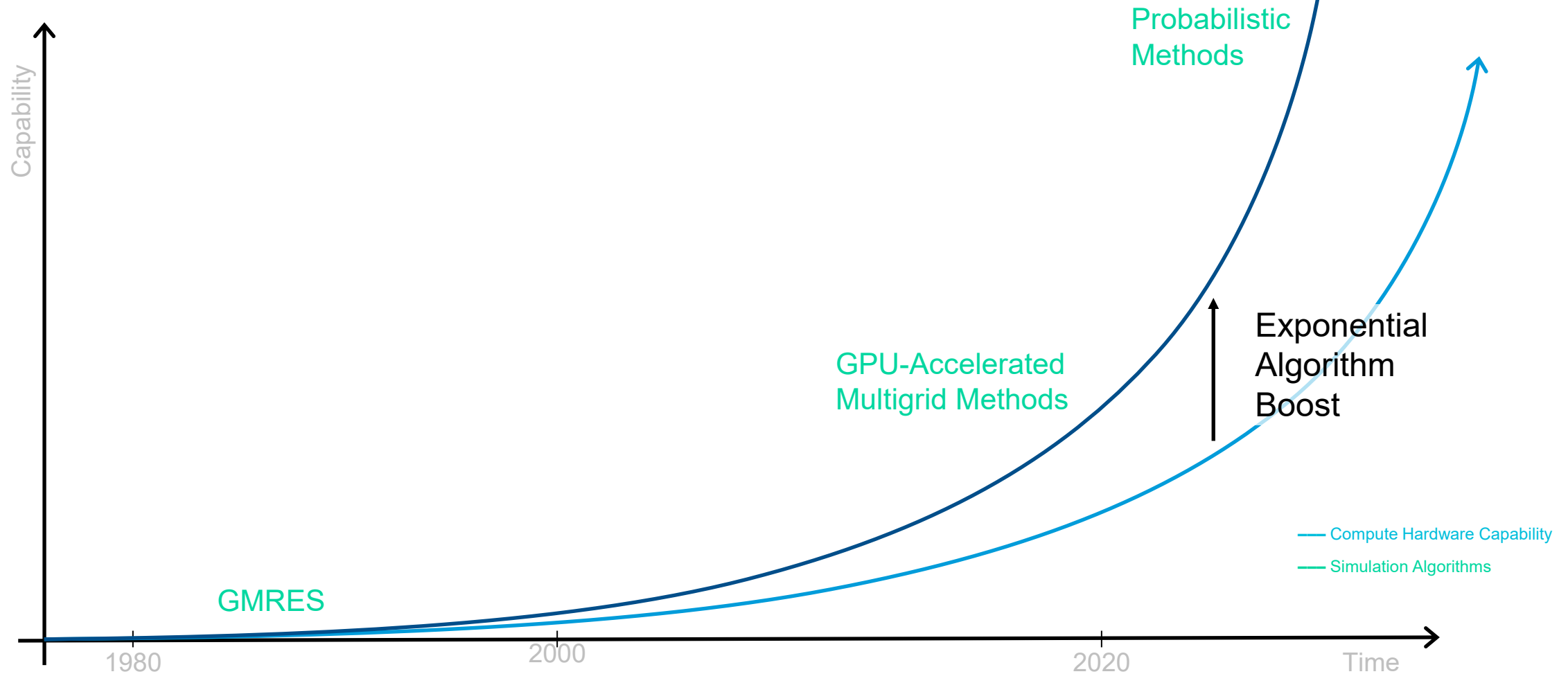
Source: John Shalf (2020): The future of computing beyond Moore's Law, Philosophical Transactions of the Royal Society A

From human computers to autonomous computational engineers

Algorithms are boosting predictive capabilities on top.



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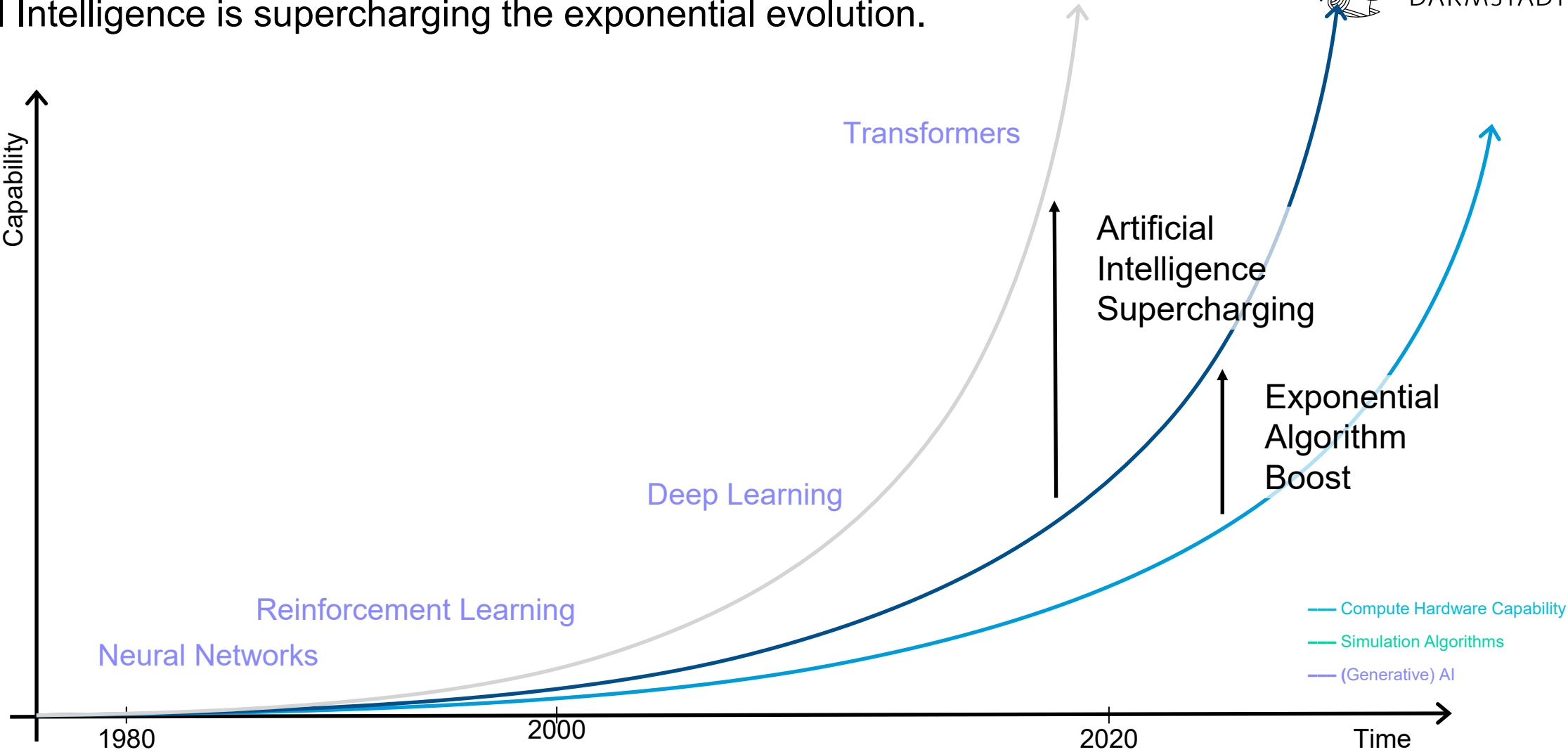
Source: U Rüde, K Willcox, L Curfman McInnes, H De Sterck (2018): Research and Education in Computational Science and Engineering, SIAM Review

From human computers to autonomous computational engineers

Artificial Intelligence is supercharging the exponential evolution.



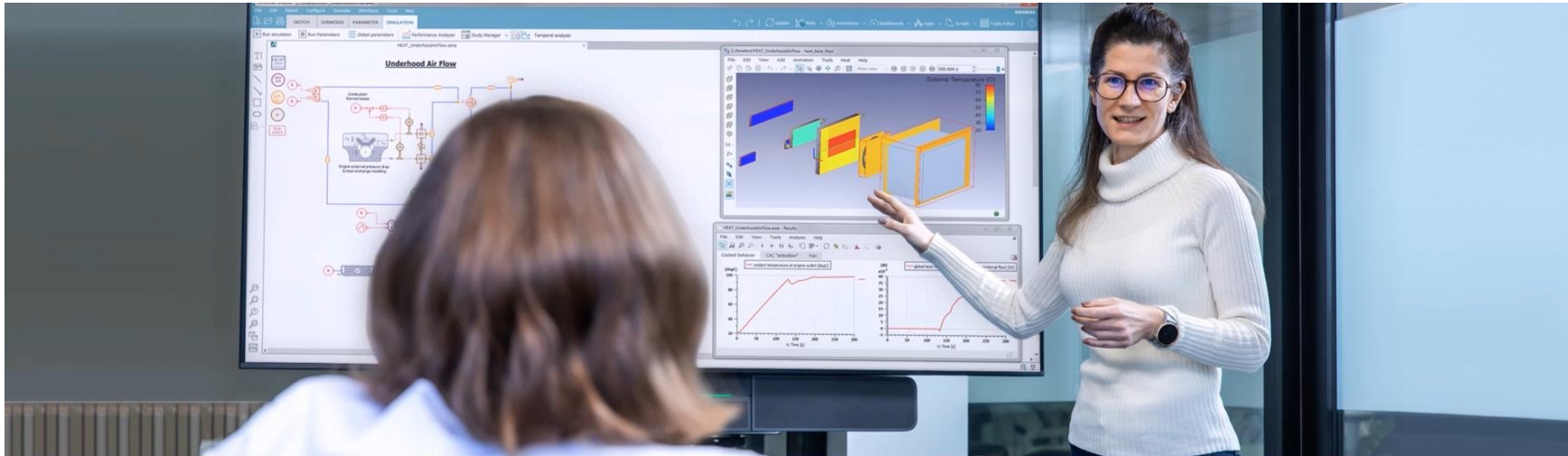
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From human computers to autonomous computational engineers



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Courtesy to Siemens AG

"Hi Simcenter" - Amesim Demo

System Modeling and Simulation in Amesim

SIEMENS



From human computers to autonomous computational engineers



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Source: [Siemens unveils breakthrough innovations in industrial AI and digital twin technologies at CES 2025](#), YouTube



1

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2

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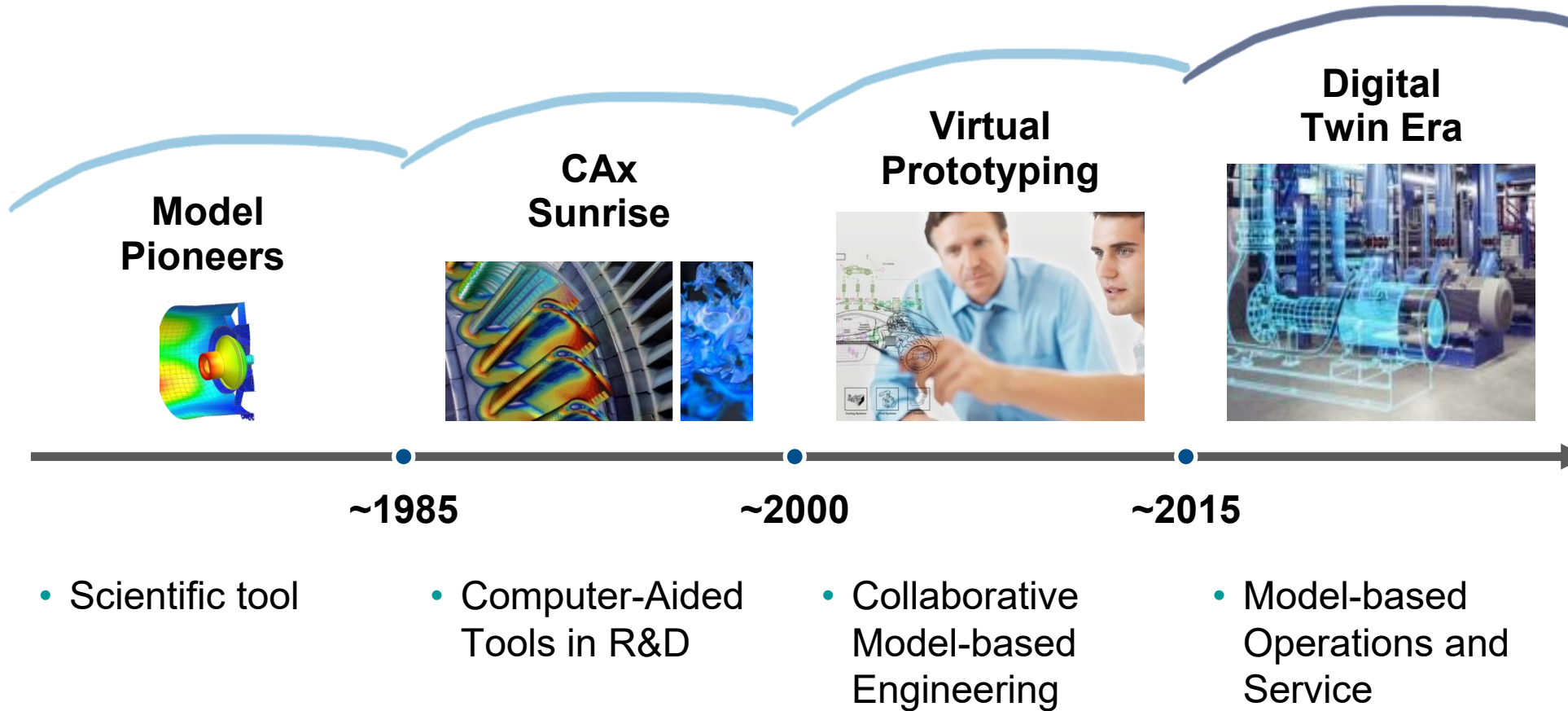
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Examples

Digital Twin – A new age of computational paradigms



The Digital Twin

In a nutshell



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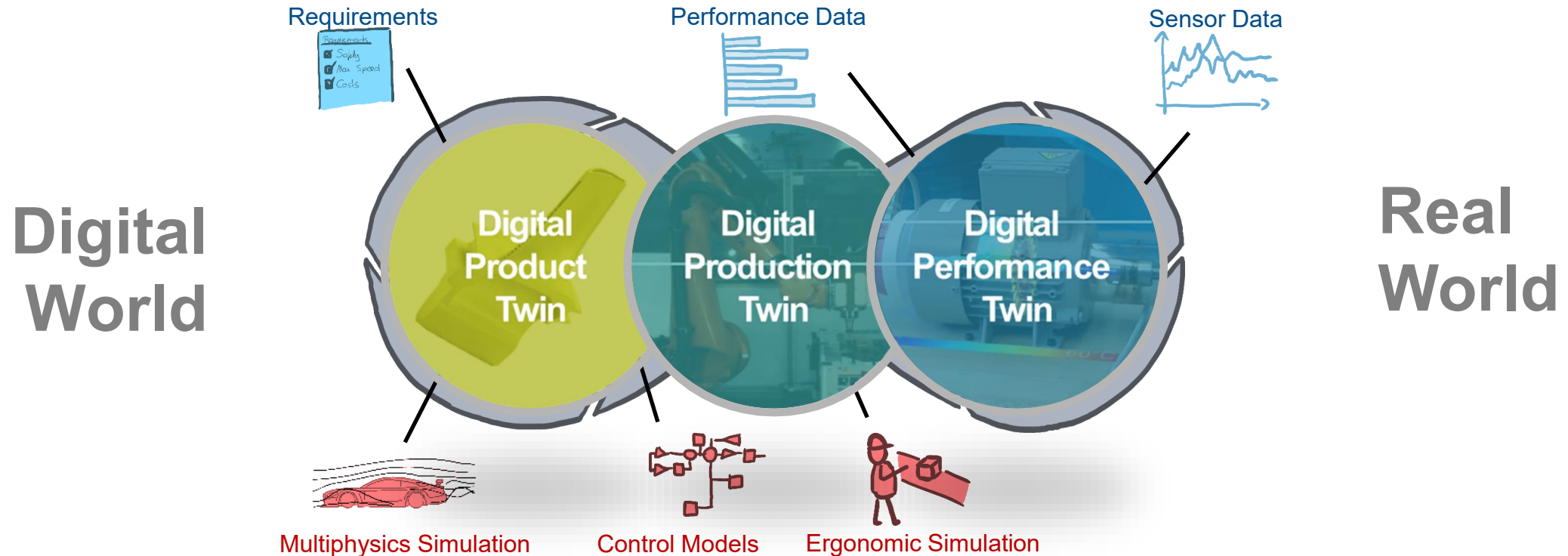


// A comprehensive **set of digital models accepted as full substitutes** for reality to understand, predict, and optimize the physical counterpart's performance characteristics for **specific purposes**. The **bidirectional interaction** between the digital and the real is central to the digital twin.

The Digital Twin

A Zoo of Models and Information

Descriptive Digital Twin



Prescriptive Digital Twin

Machine Learning for Digital Models

Dynamical System Identification (“Learning Dynamics from Observations”)

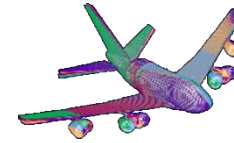
$$\frac{d}{dt} \mathbf{x} = f(\mathbf{x}, t, \mathbf{u})$$

Diagram illustrating the dynamical system equation $\frac{d}{dt} \mathbf{x} = f(\mathbf{x}, t, \mathbf{u})$ with annotations:

- State**: Points to $\mathbf{x}$
- (external, control) Actuation**: Points to $\mathbf{u}$
- Dynamics**: Points to f , with the text "System of Differential Equations" below it.
- Time**: Points to t



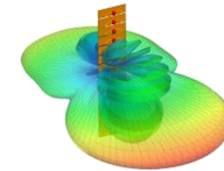
Motion



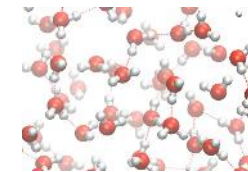
Mechanics



Fluid
Dynamics



Electro-
dynamics



Chemistry

...

Major categories of Digital Twins for Operation and Service

Category	Use this application when you want to...	Examples
 Virtual Testing & Commissioning	... prepare for how your asset or system would interact with other assets, systems, or people.	• Testing of automation by virtual commissioning • Testing new control strategies on gas turbines • Operator training
 Virtual Sensing	...measure something in your asset or system, but it isn't feasible to put a sensor there.	• Temperature inside electric rotor • Pressure distribution inside a gas turbine • Free-flow inside a sewage network
 Diagnosis & Identification	...know why your asset or system is behaving the way it is.	• Unbalance detection of large rotors • Leakage detection in a water distribution network • Predictive maintenance for machine tools
 Performance Prediction	...know how your asset or system might behave in future operation.	• Remaining useful lifetime of electric motors • Monitoring of coking in steam cracking furnaces • Movement of people in emergencies
 Performance Optimization	...inform actions on how to control the asset or system (with or without a Human-in-the-Loop).	• Model predictive control of a chemical reactor • Pump schedule optimization of oil pipelines • Operating point setting of catalyst modules

Source: D. Hartmann (2021): [Real-time Digital Twins](#).

The (Executable) Digital Twin for Operations

Virtual Sensing – Thermal Management of Electric Motors



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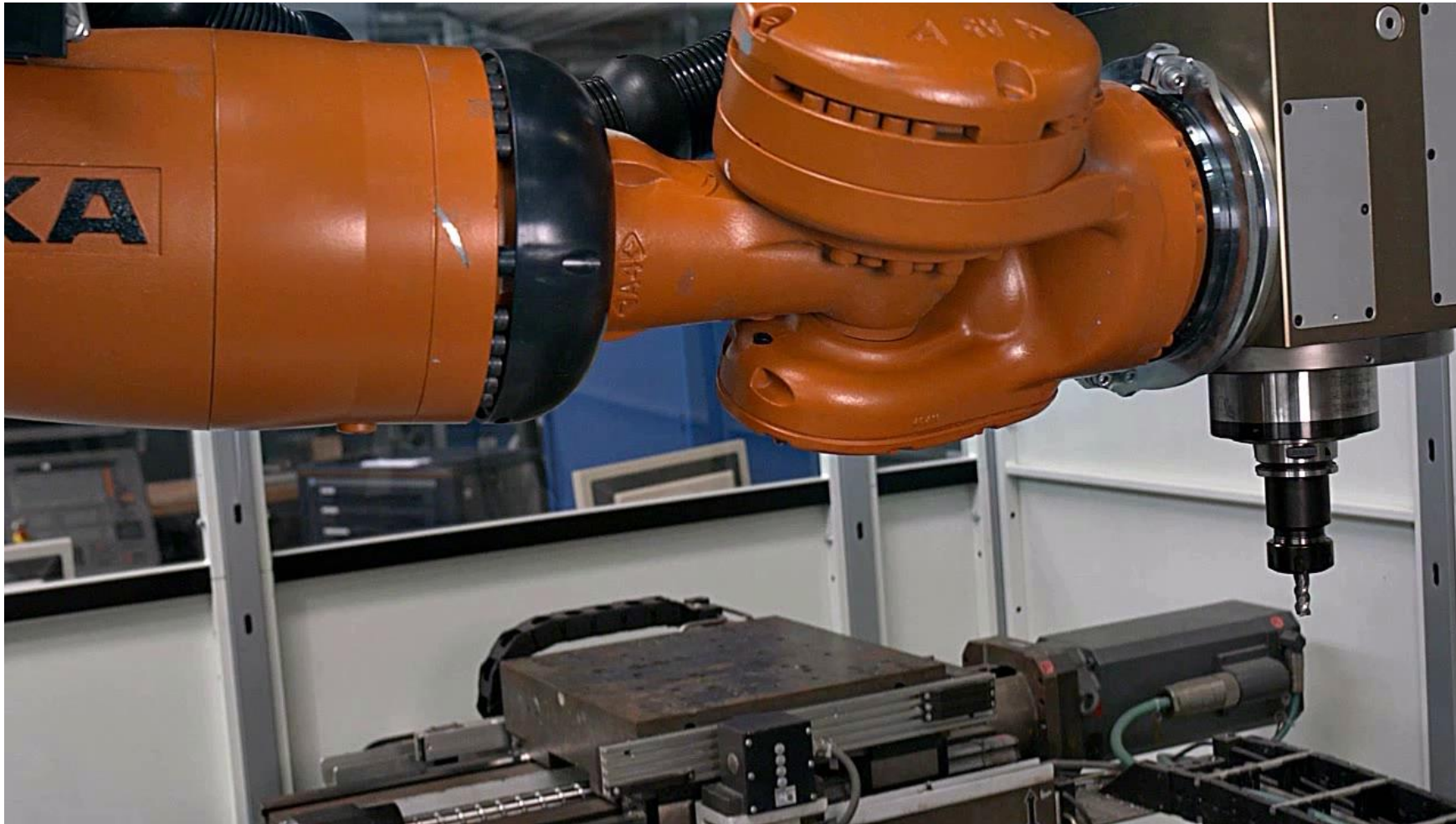


The (Executable) Digital Twin for Operations

Performance Optimization – Improving Accuracy Robotic Milling



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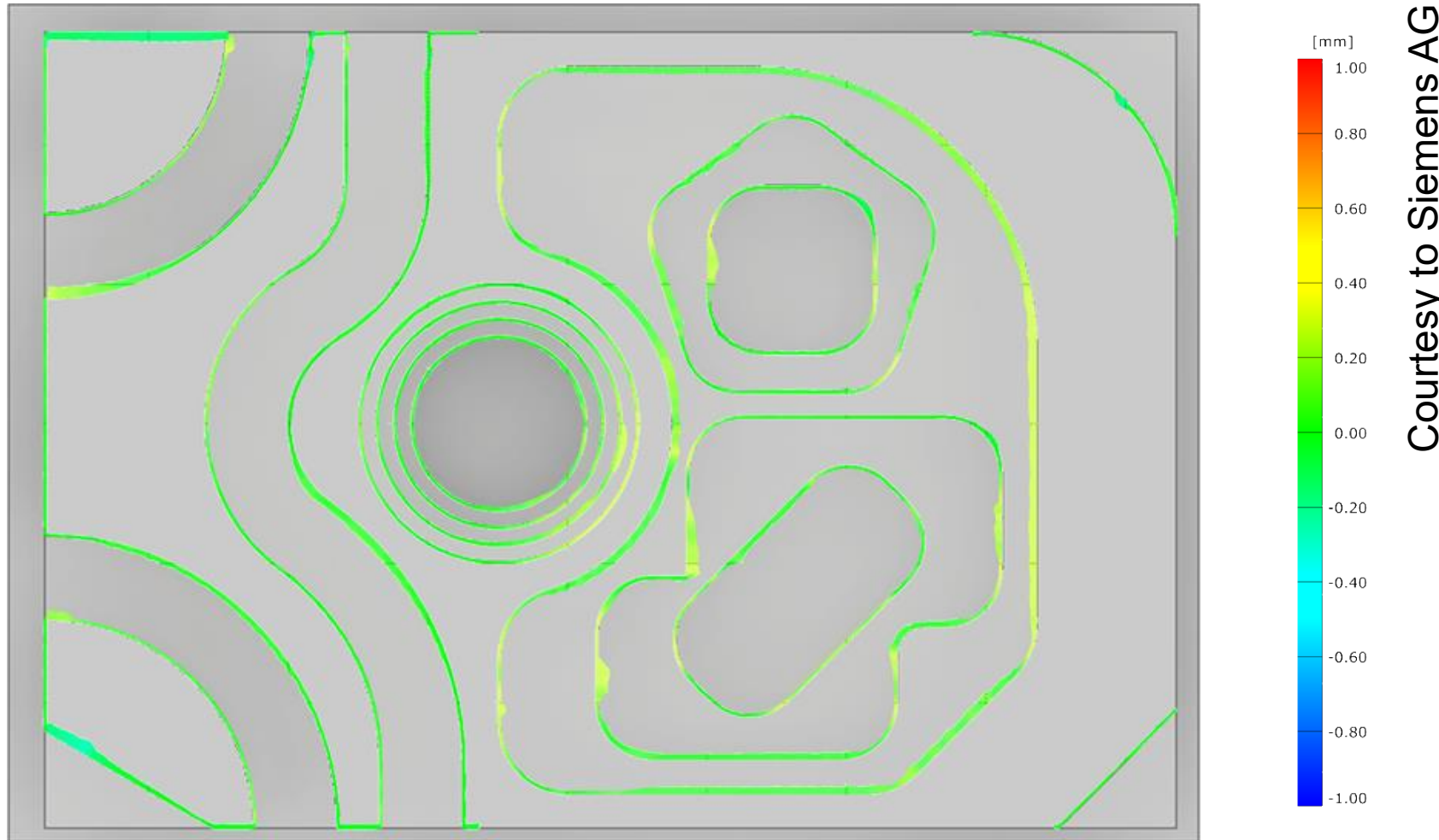


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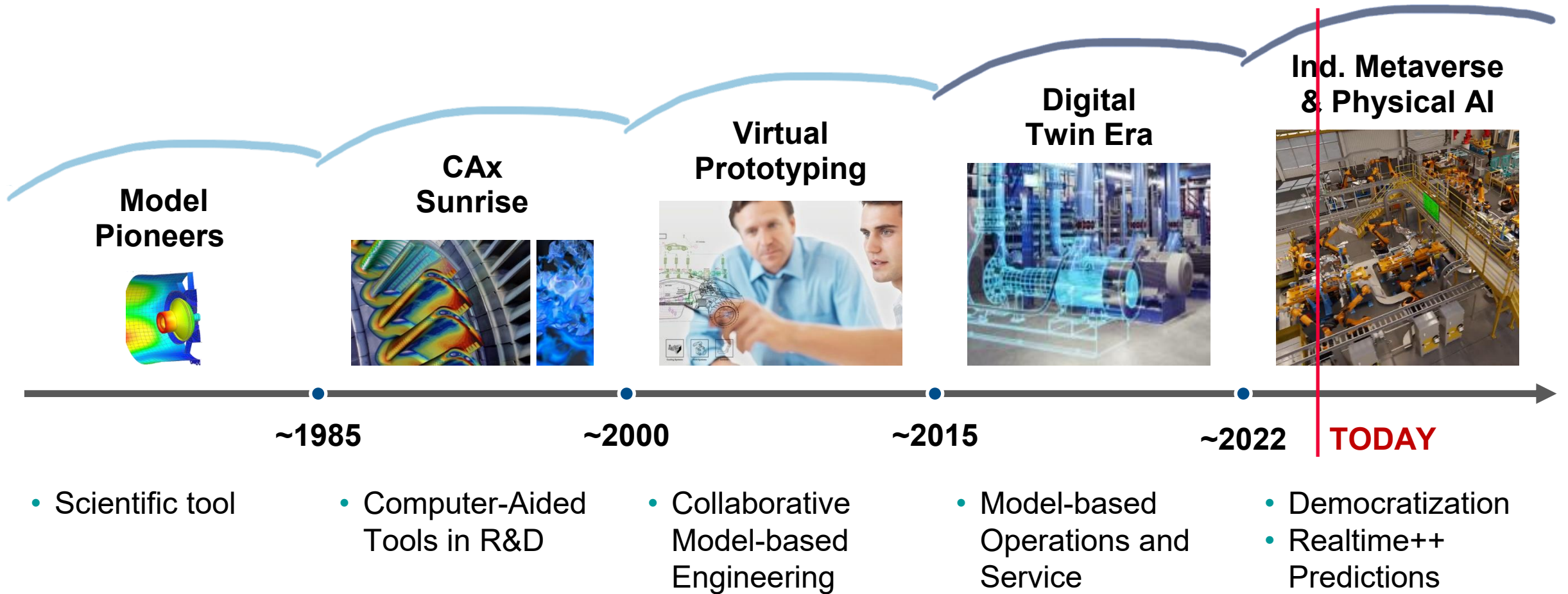


The (Executable) Digital Twin for Operations

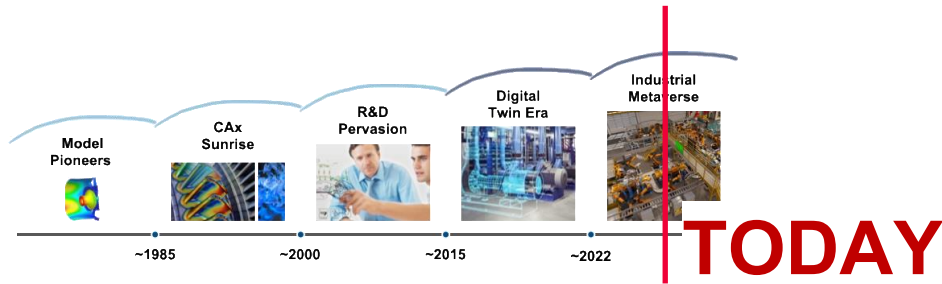
Performance Optimization – Improving Accuracy Robotic Milling



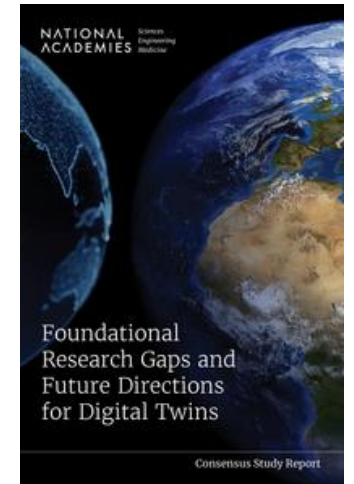
Digital Twin - *A new age of computational paradigms*



Digital Twin – *State of Industrial Adoption Today*



“Despite [...] examples of digital twins providing practical impact and value [...] publicity around digital twins [...] currently outweighs the evidence base of success.”



Source: National Academies (2024): [Foundational Research Gaps and Future Directions for Digital Twins](#).



1

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2

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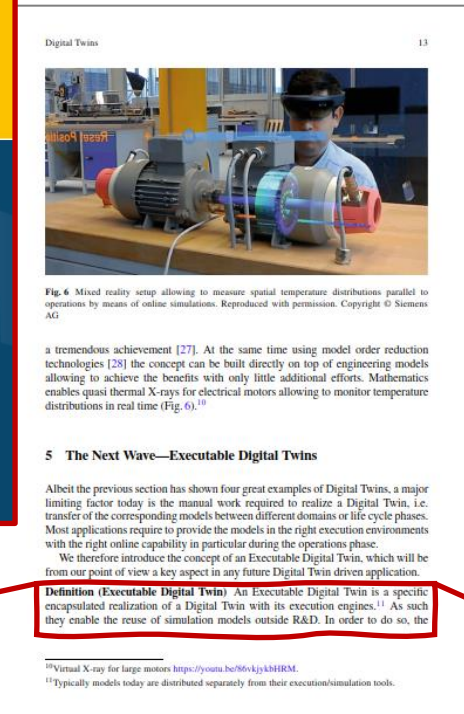
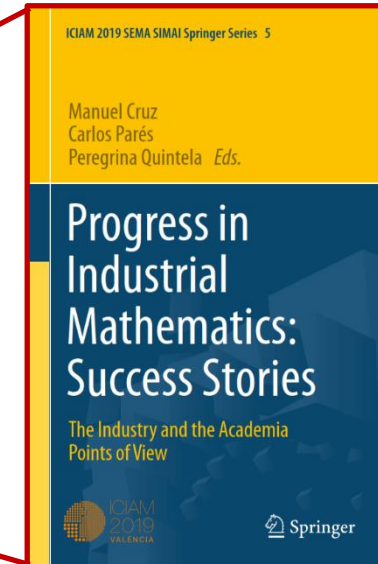
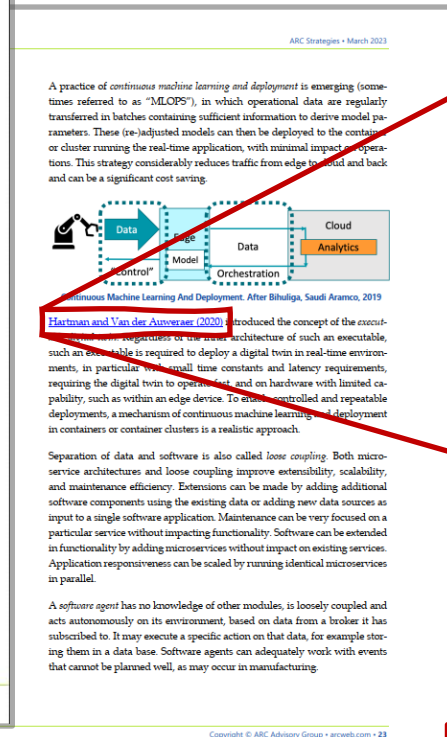
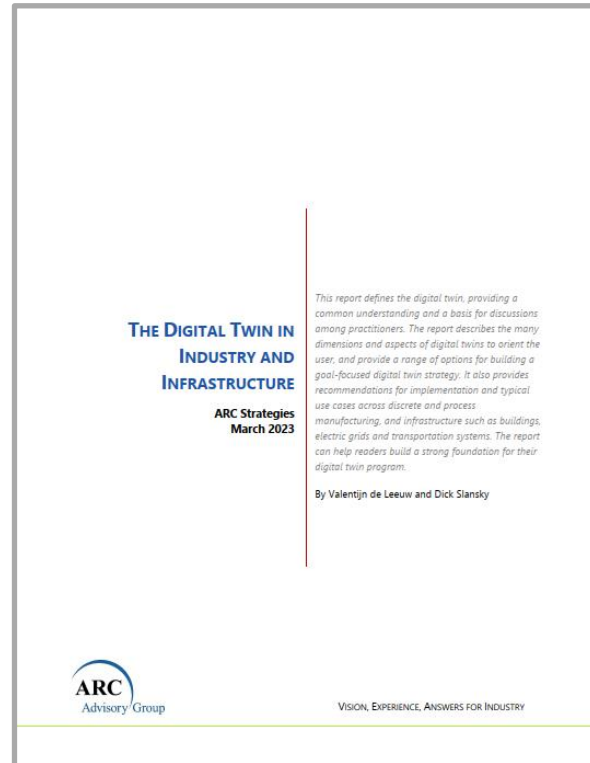
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Real-time Algorithms

4

Examples

The “first” math paper quoted in an Industry Analyst paper.



Definition (Executable Digital Twin) An Executable Digital Twin is a specific encapsulated realization of a Digital Twin with its execution engines.¹¹ As such they enable the reuse of simulation models outside R&D. In order to do so, the ...

Sources: V. De Leeuw, D. Slansky (2023): [The Digital Twin in Industry and Infrastructure](#), ARC Advisory Group Industry Report
D. Hartmann, H. van der Auweraer (2020); [Digital Twins](#), Progress in Industrial Mathematics: Success Stories

How to achieve Real-time simulation?

Core Concepts

Approach 1



Choose optimal algorithms and fully exploit hardware

- highly efficient simple (matrix-free) discretization
 - Optimal Geometric multi-grid method
 - Exploit massively parallel hardware
- Highly efficient and generalizable finite element solver

Approach 2

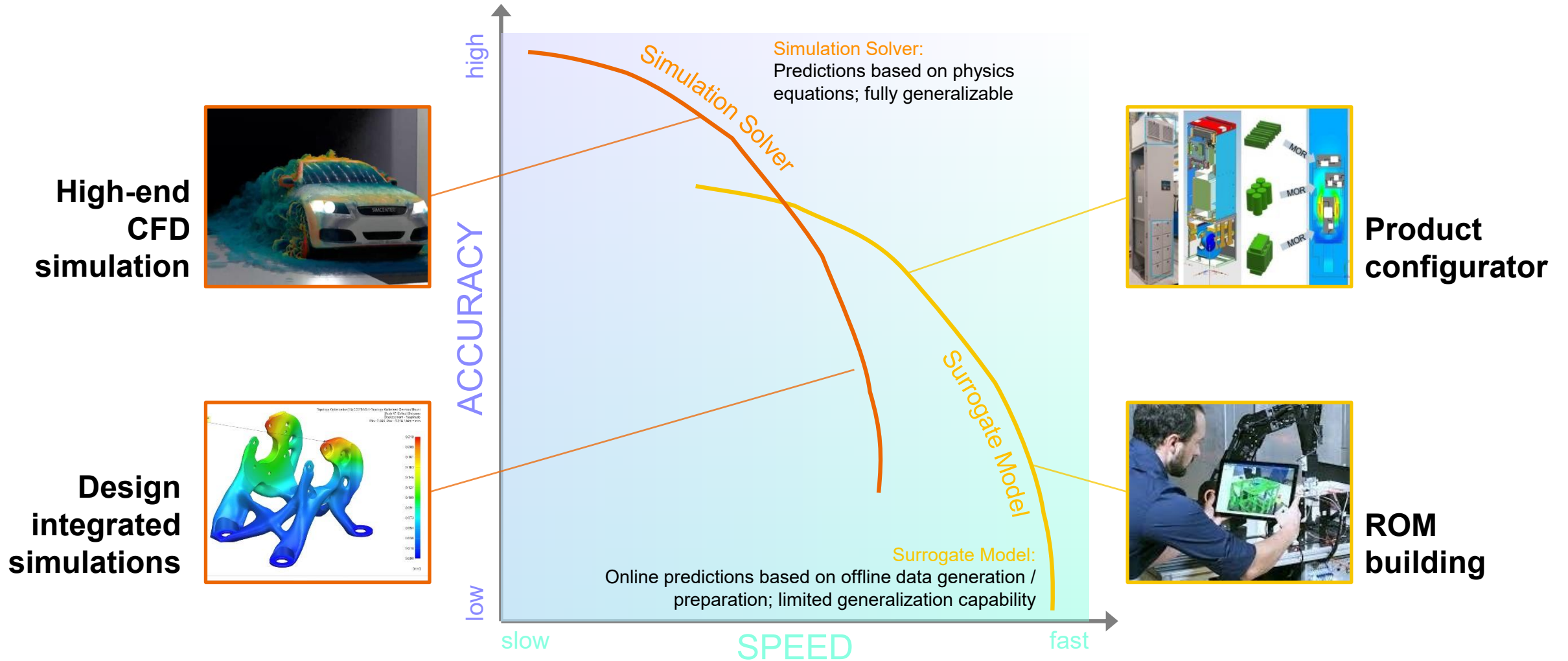


Choose an optimal representation

- Optimal basis function leads with limited number of variables (Model Order Reduction)
 - Requires, split in an offline and online phase to compute appropriate basis function
- Highly efficient solver, but limited to a specific application

How to achieve Real-time simulation?

Core Concepts



Example: Efficient Linear Algebra enables Interactive and Realtime Engineering



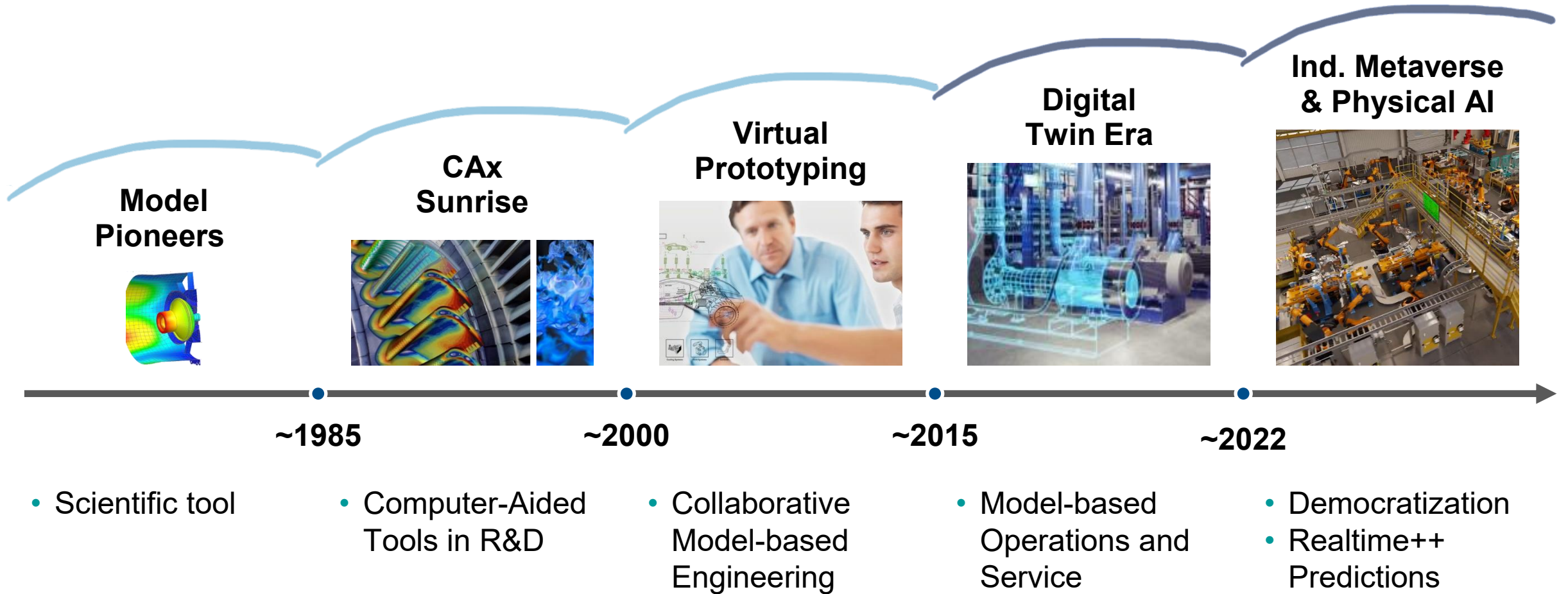
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Digital Twin - *A new age of computational paradigms*





1

Digital Engineering

2

Digital Twins

3

Real-time Algorithms

4

Examples

Real-Time Algorithms for Digital Twins

Exercise Examples

T at t = 0.0 s (c = -0.45, h = 1.17)

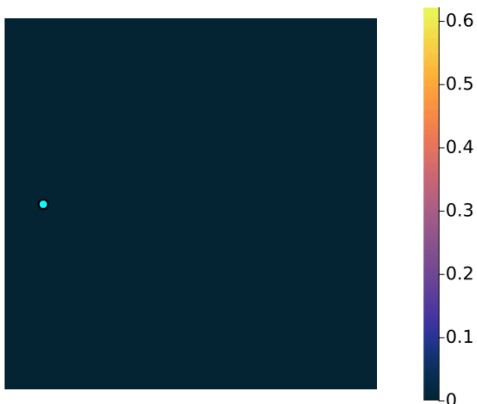


“Electronics Cooling”

$$\partial_t T = \Delta T + c(t)q(x)$$

- Linear PDE
- Transient cooling and heating
- Geometry via pictures
- The poster-child for ROM

T at t = 0.0 s



“Selective Laser Melting”

$$\partial_t T = \Delta T + q(x, t)$$

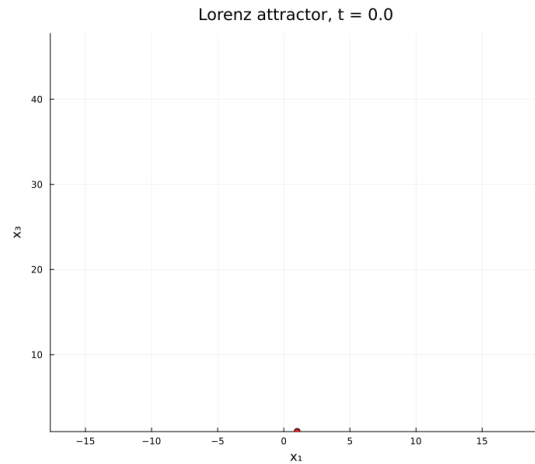
- Linear PDE
- Heating via a moving heat source
- The nightmare for ROM

Real-Time Algorithms for Digital Twins

Exercise Examples

Lorenz System

$$\begin{aligned}\partial_t x &= \sigma(y - x) \\ \partial_t y &= x(\rho - z) - y \\ \partial_t z &= xy - \beta z\end{aligned}$$



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Contact



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