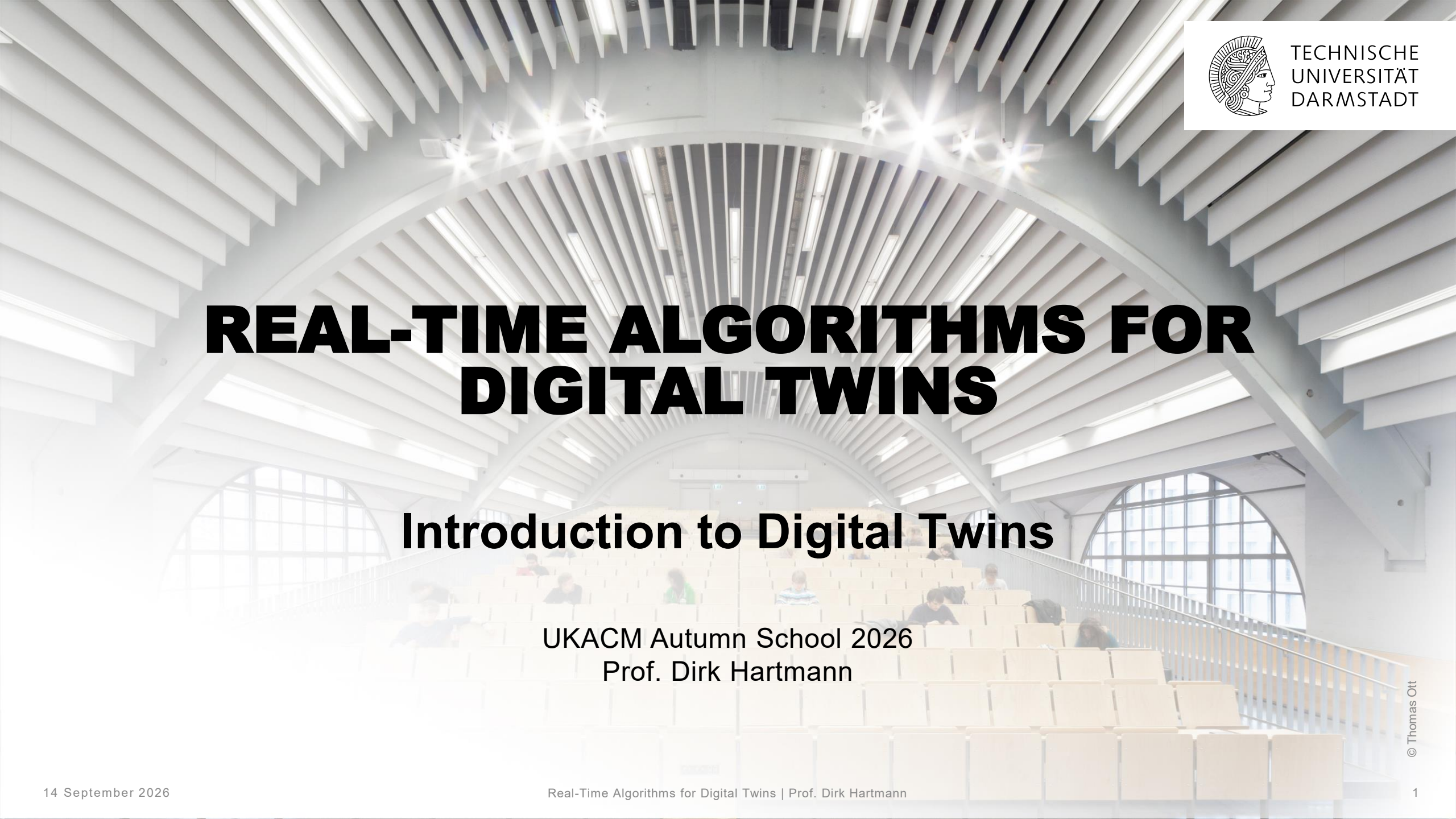

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REAL-TIME ALGORITHMS FOR DIGITAL TWINS

Introduction to Digital Twins

UKACM Autumn School 2026
Prof. Dirk Hartmann

Real-Time Algorithms for Digital Twins
Dirk Hartmann

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Dr. Dirk Hartmann

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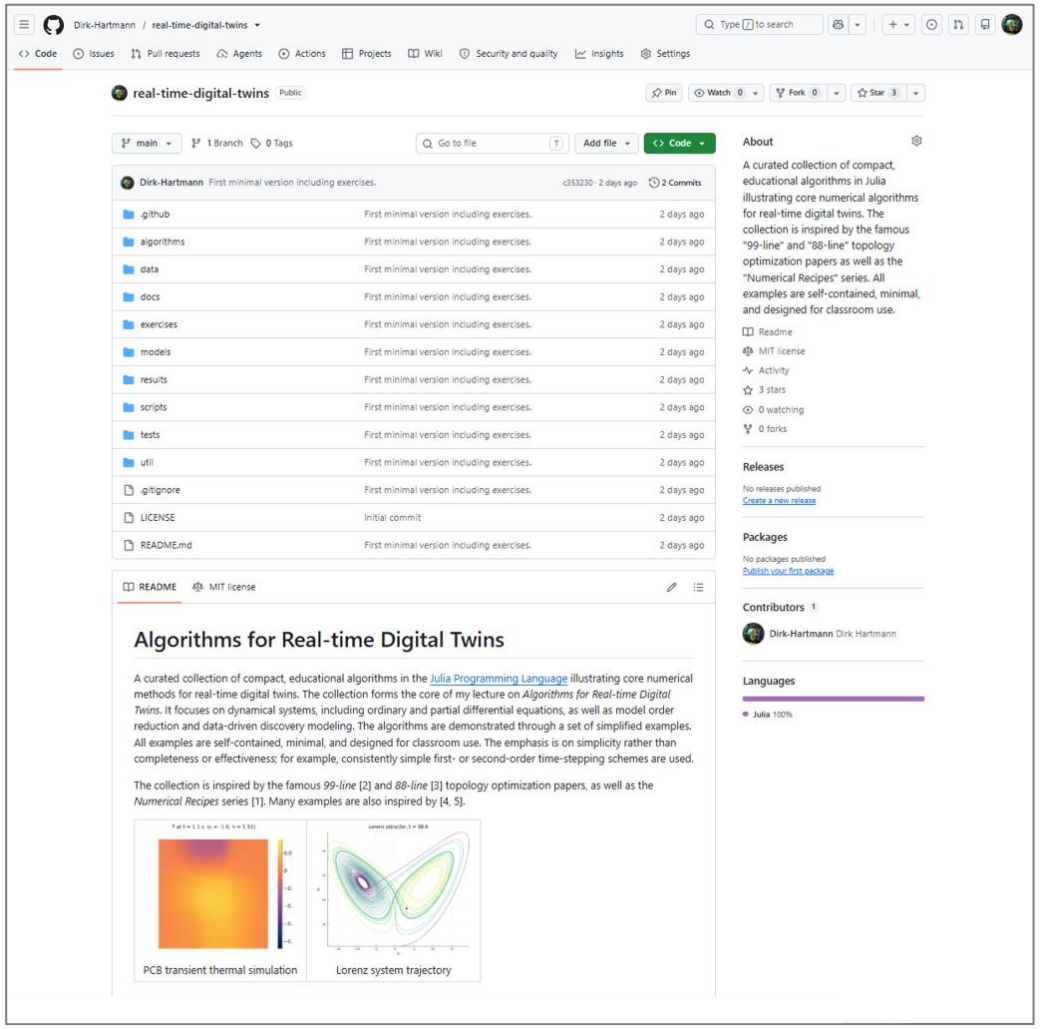
Homepage: https://dirk-hartmann.github.io
TU Darmstadt: https://www.temf.tu-darmstadt.de/digitaltwin
LinkedIn: https://www.linkedin.com/in/dirkhartmann

The Art of the Possible – Blog: https://blogs.sw.siemens.com/art-of-the-possible

Real-Time Algorithms for Digital Twins
GitHub Repository



Please leave a ★, if you like it!

https://github.com/Dirk-Hartmann/real-time-digital-twins

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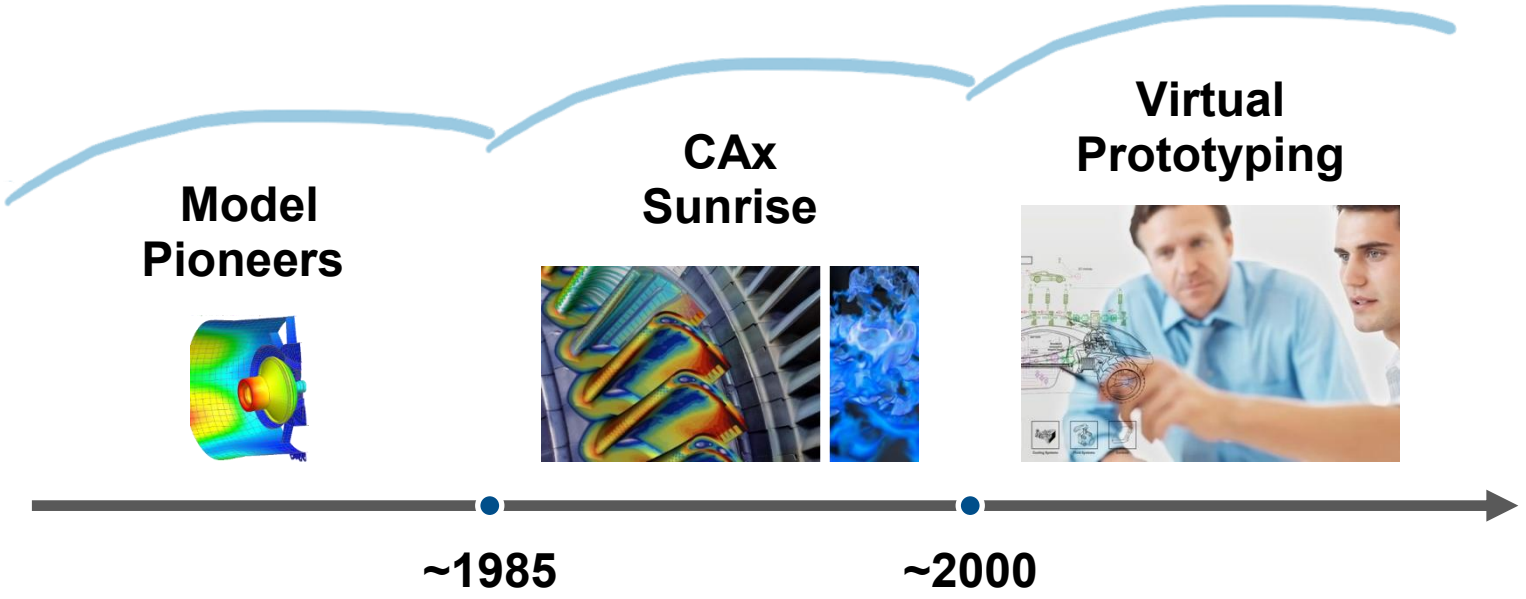
1 Digital Engineering

2 Digital Twins

3 Real-time Algorithms

4 Examples

Digital Twin – *An old story!*



- Scientific tool
- Computer-Aided Tools in R&D
- Collaborative Model-based Engineering

CAx: Computer Aided Design, Engineering, & Manufacturing

From human computers to autonomous computational engineers

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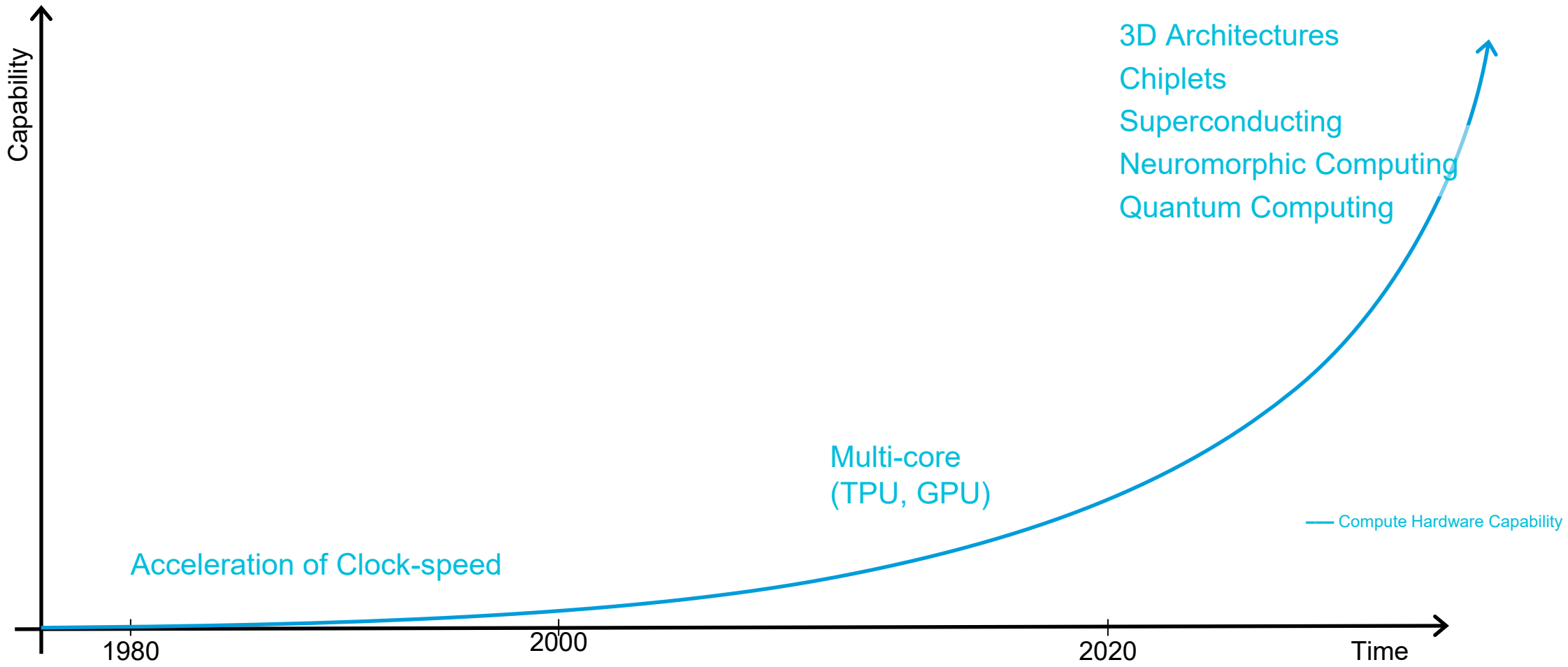
Anthony Greicius (2016): When Computers Were Human. NASA Blog

From human computers to autonomous computational engineers

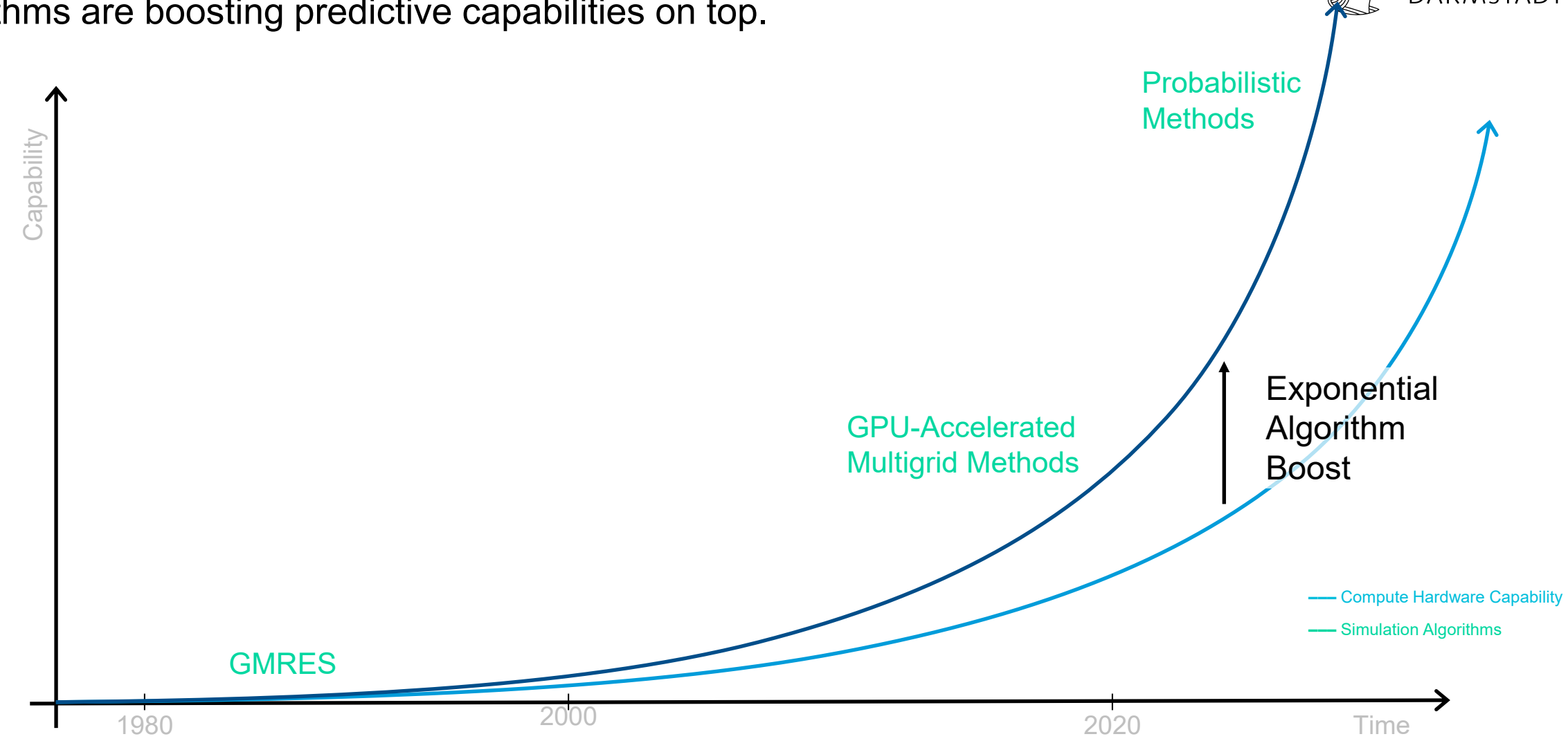
Electronics is evolving delivering more than Moore's law.



Source: John Shalf (2020): The future of computing beyond Moore's Law, Philosophical Transactions of the Royal Society A

From human computers to autonomous computational engineers

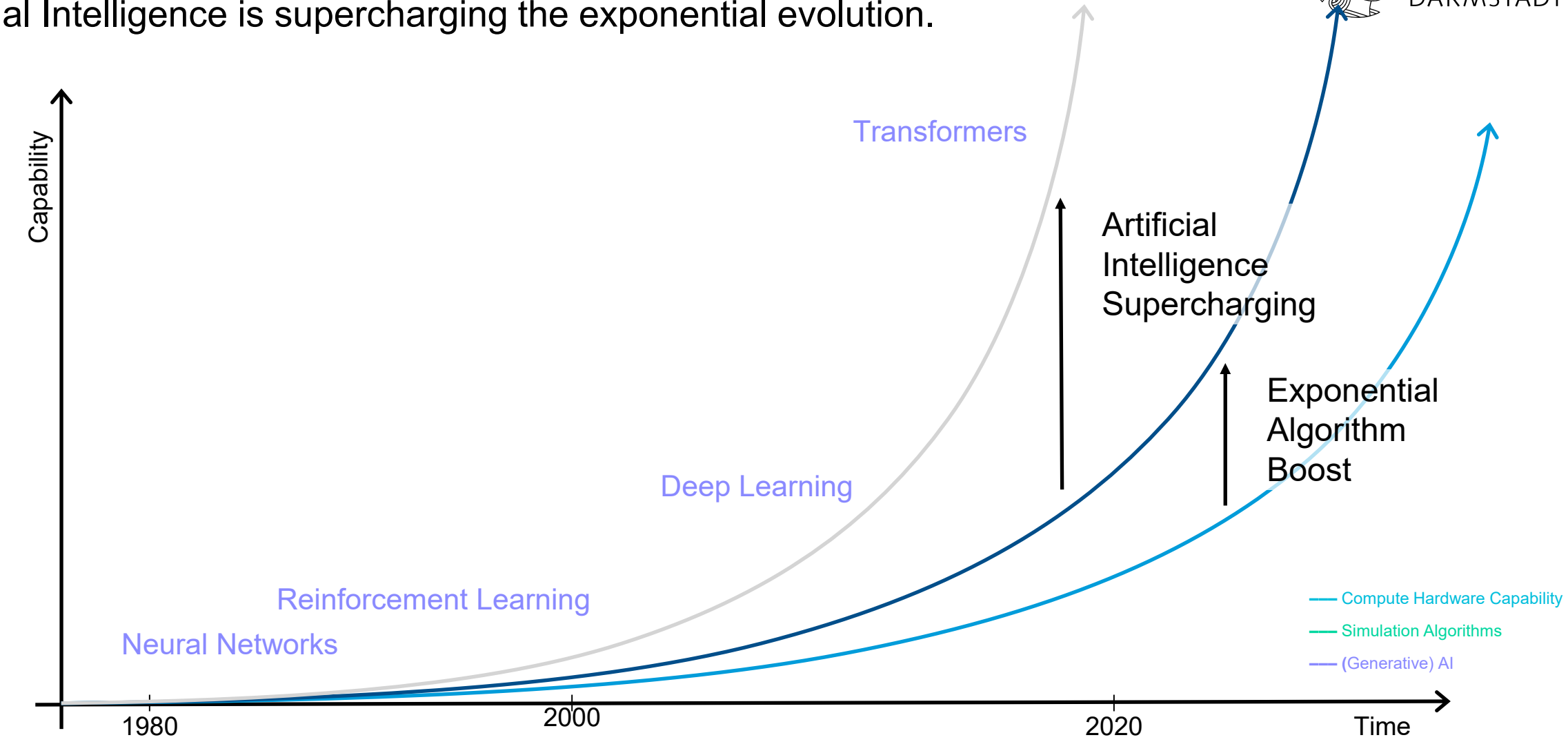
Algorithms are boosting predictive capabilities on top.



Source: U Rüde, K Willcox, L Curfman McInnes, H De Sterck (2018): Research and Education in Computational Science and Engineering, SIAM Review

From human computers to autonomous computational engineers

Artificial Intelligence is supercharging the exponential evolution.



From human computers to autonomous computational engineers

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Courtesy to Siemens AG

From human computers to autonomous computational engineers

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Courtesy to Siemens AG

Source: Siemens unveils breakthrough innovations in industrial AI and digital twin technologies at CES 2025, YouTube

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4 Examples

Digital Twin – *A new age of computational paradigms*

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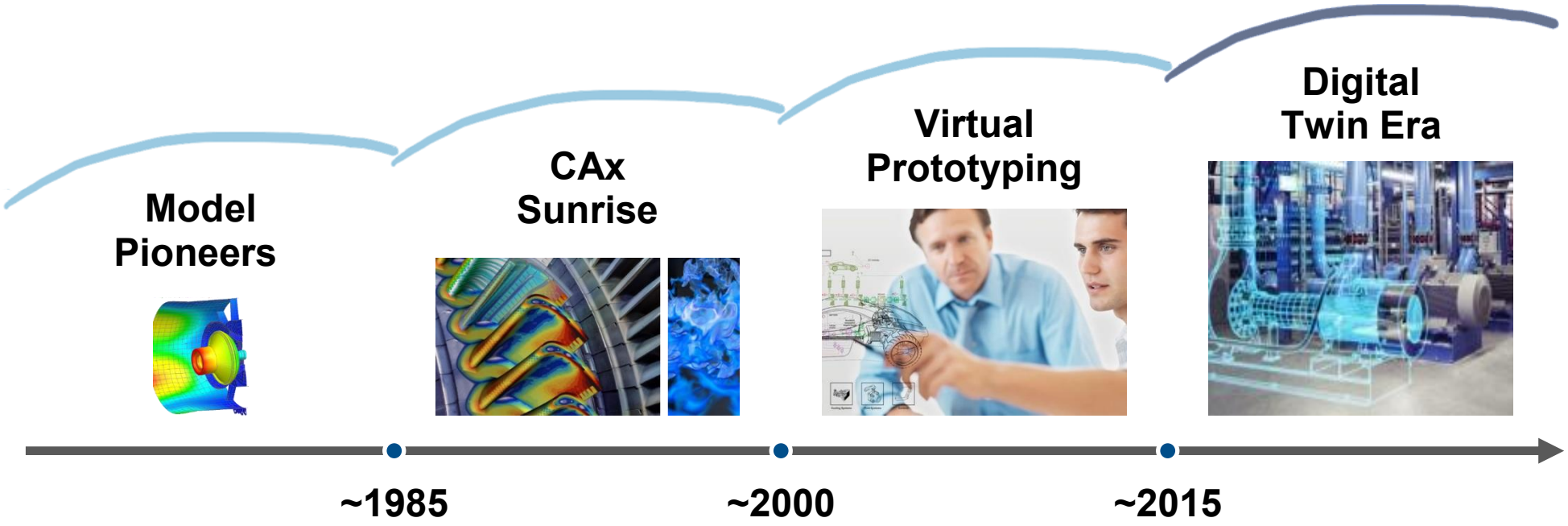

- Scientific tool
- Computer-Aided Tools in R&D
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CAx: Computer Aided Design, Engineering, & Manufacturing

The Digital Twin
In a nutshell

Courtesy to Siemens AG

The Digital Twin
In a nutshell

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> A comprehensive set of digital models accepted as full substitutes for reality to understand, predict, and optimize the physical counterpart's performance characteristics for specific purposes. The bidirectional interaction between the digital and the real is central to the digital twin.

The Digital Twin
A Zoo of Models and Information

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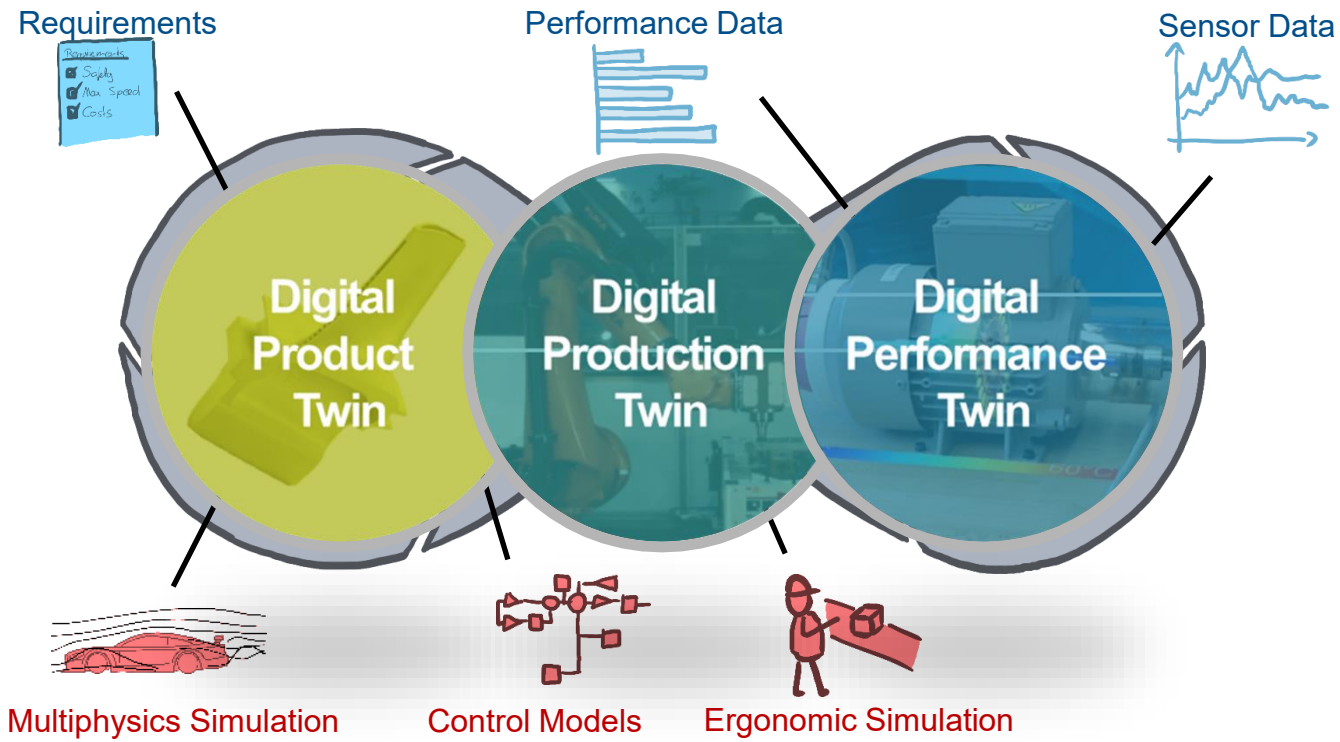
Descriptive Digital Twin

Digital World



Real World

Prescriptive Digital Twin

Machine Learning for Digital Models

Dynamical System Identification (“Learning Dynamics from Observations”)

$$\frac{d}{dt}\boldsymbol{x} = f\,(\boldsymbol{x}, t, \boldsymbol{u})$$

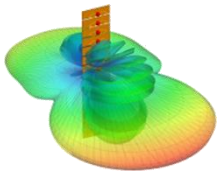
State

(external, control) **Actuation**

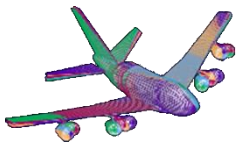
Dynamics
System of Differential Equations

Time

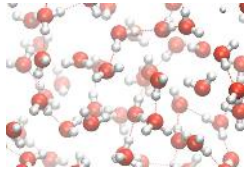
Motion



Electro-dynamics



Mechanics



Chemistry

Fluid Dynamics

...

Major categories of Digital Twins for Operation and Service

Category	Use this application when you want to…	Examples
 **Virtual Testing & Commissioning**	… prepare for how your asset or system would interact with other assets, systems, or people.	• Testing of automation by virtual commissioning • Testing new control strategies on gas turbines • Operator training
 **Virtual Sensing**	…measure something in your asset or system, but it isn't feasible to put a sensor there.	• Temperature inside electric rotor • Pressure distribution inside a gas turbine • Free-flow inside a sewage network
 **Diagnosis & Identification**	…know why your asset or system is behaving the way it is.	• Unbalance detection of large rotors • Leakage detection in a water distribution network • Predictive maintenance for machine tools
 **Performance Prediction**	…know how your asset or system might behave in future operation.	• Remaining useful lifetime of electric motors • Monitoring of coking in steam cracking furnaces • Movement of people in emergencies
 **Performance Optimization**	…inform actions on how to control the asset or system (with or without a Human-in-the-Loop).	• Model predictive control of a chemical reactor • Pump schedule optimization of oil pipelines • Operating point setting of catalyst modules

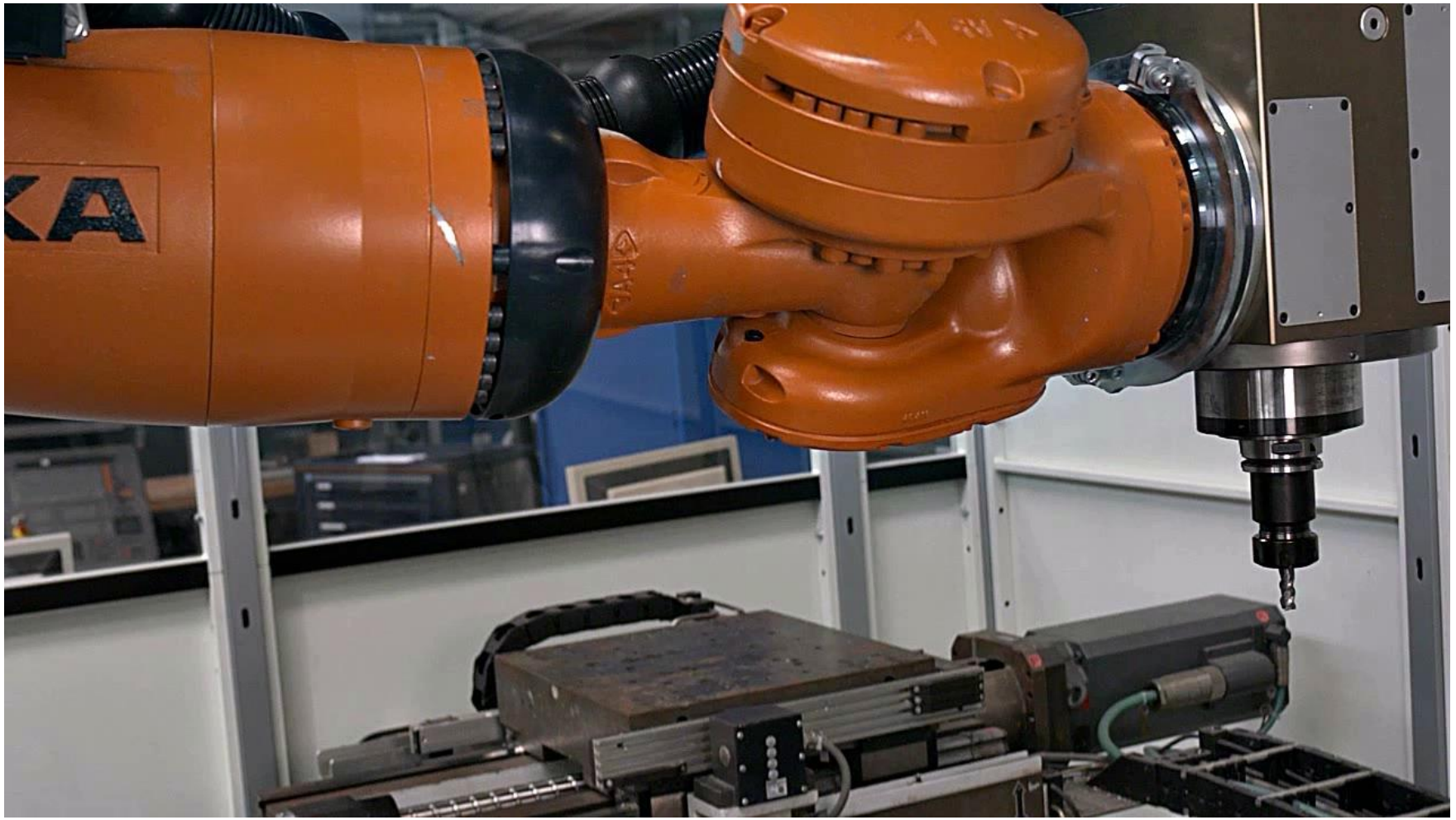
Source: D. Hartmann (2021): Real-time Digital Twins.

The (Executable) Digital Twin for Operations
Virtual Sensing – Thermal Management of Electric Motors

Courtesy to Siemens AG

The (Executable) Digital Twin for Operations

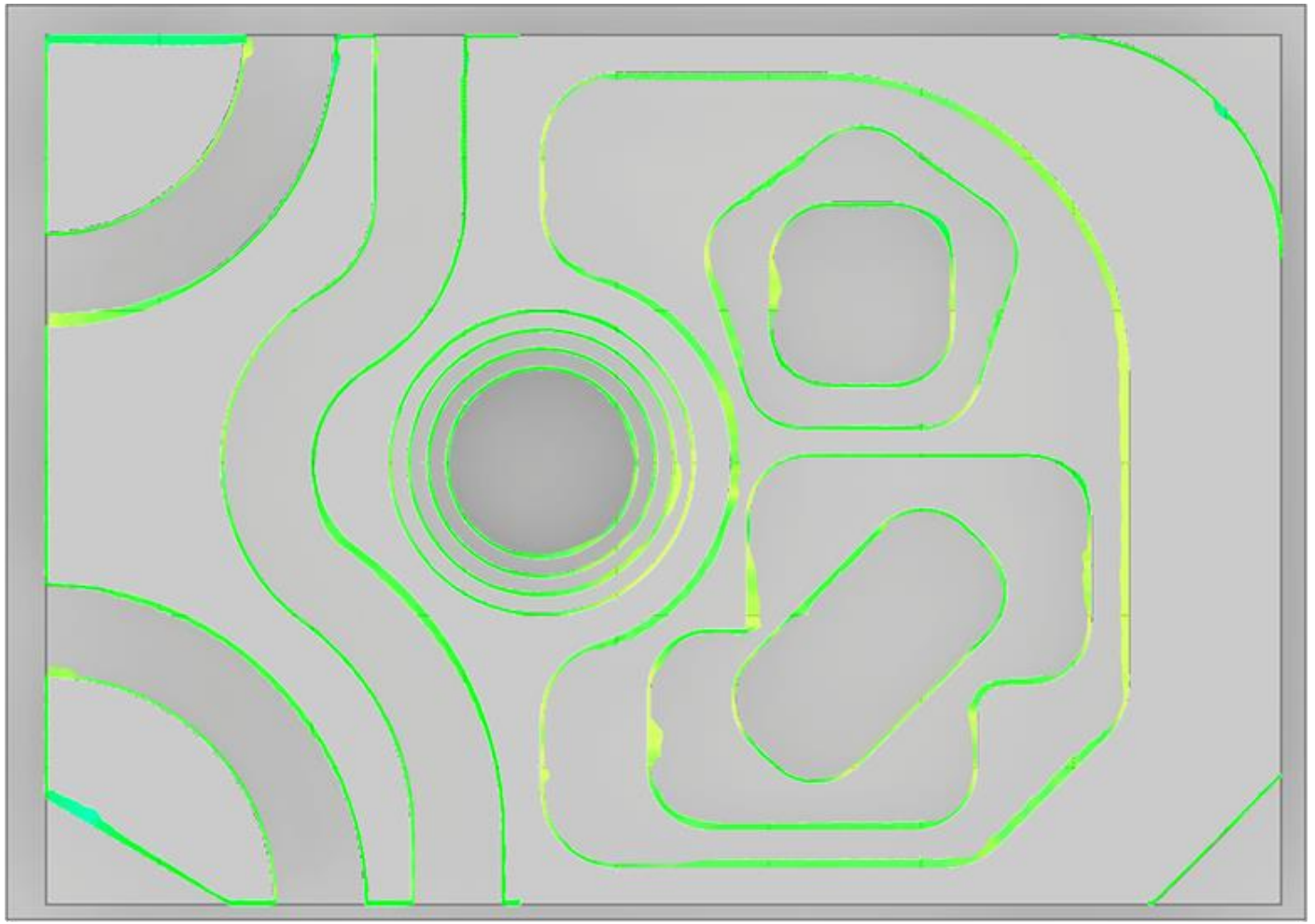
Performance Optimization – Improving Accuracy Robotic Milling

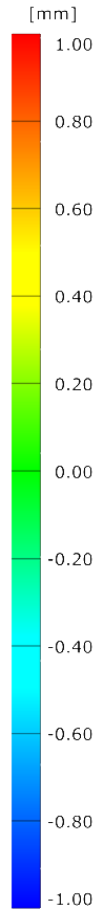


Courtesy to Siemens AG

The (Executable) Digital Twin for Operations

Performance Optimization – Improving Accuracy Robotic Milling





Courtesy to Siemens AG

Digital Twin - *A new age of computational paradigms*

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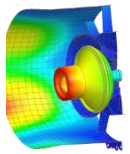
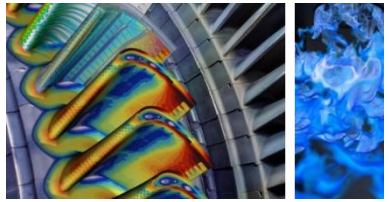
Model Pioneers



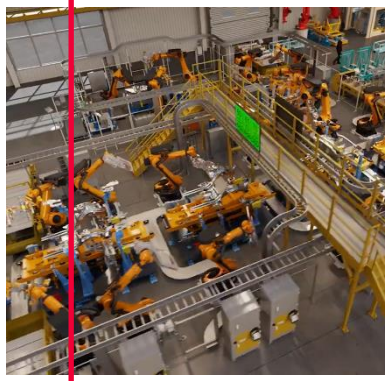
CAx Sunrise



Virtual Prototyping

Digital Twin Era

Ind. Metaverse & Physical AI

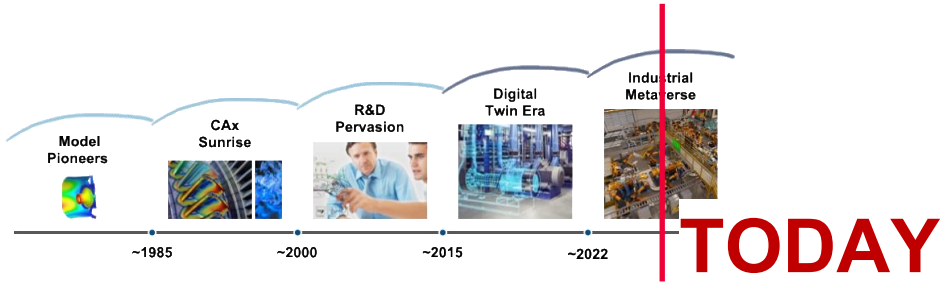


~1985 | **~2000** | **~2015** | **~2022** | **TODAY**

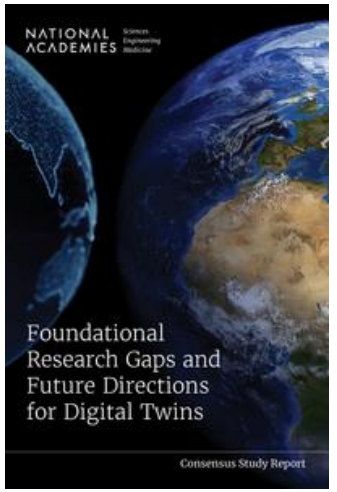
- Scientific tool
- Computer-Aided Tools in R&D
- Collaborative Model-based Engineering
- Model-based Operations and Service
- Democratization
- Realtime++ Predictions

CAx: Computer Aided Design, Engineering, & Manufacturing

Digital Twin – *State of Industrial Adoption Today*



"Despite […] examples of digital twins providing practical impact and value […] publicity around digital twins […] currently outweighs the evidence base of success."



Source: National Academies (2024): Foundational Research Gaps and Future Directions for Digital Twins.

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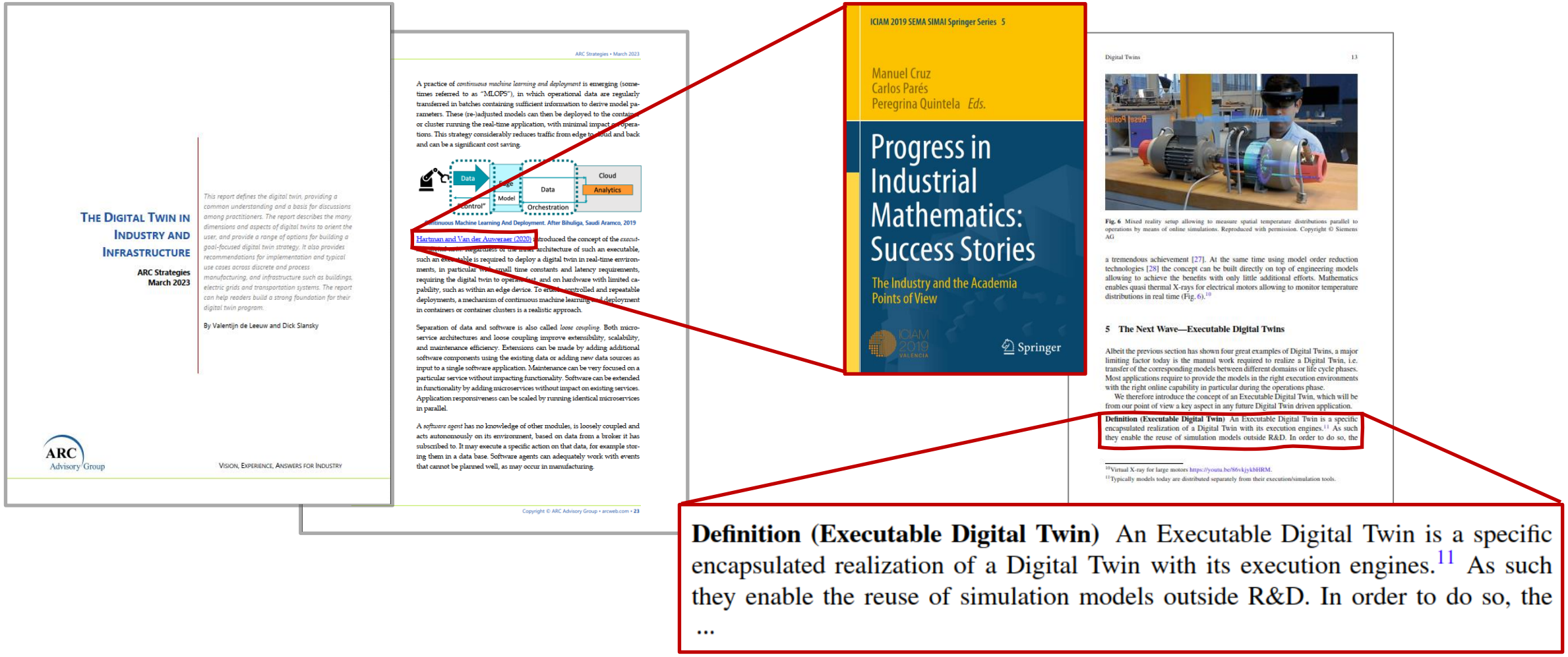
1 Digital Engineering

2 Digital Twins

3 Real-time Algorithms

4 Examples

The “first” math paper quoted in an Industry Analyst paper.



Sources: V. De Leeuw, D. Slansky (2023): The Digital Twin in Industry and Infrastructure, ARC Advisory Group Industry Report
D. Hartmann, H. van der Auweraer (2020); Digital Twins, Progress in Industrial Mathematics: Success Stories

How to achieve Real-time simulation?
Core Concepts

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Approach 1

Choose optimal algorithms and fully exploit hardware

- highly efficient simple (matrix-free) discretization
- Optimal Geometric multi-grid method
- Exploit massively parallel hardware

▶ Highly efficient and generalizable finite element solver

Approach 2

Choose an optimal representation

- Optimal basis function leads with limited number of variables (Model Order Reduction)
- Requires, split in an offline and online phase to compute appropriate basis function

▶ Highly efficient solver, but limited to a specific application

How to achieve Real-time simulation?
Core Concepts

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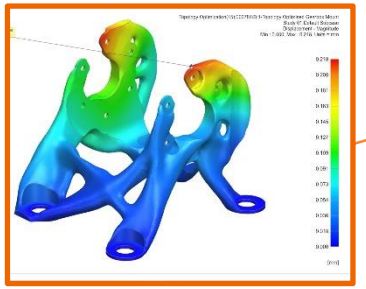
High-end CFD simulation

Design integrated simulations



Product configurator

ROM building

Example:
Efficient Linear Algebra enables Interactive and Realtime Engineering

Courtesy to Siemens AG

Digital Twin - *A new age of computational paradigms*

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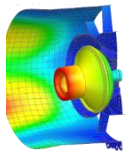
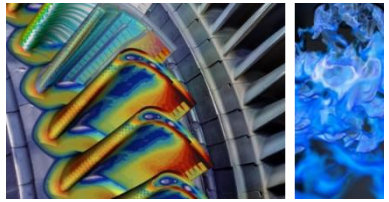
Model Pioneers



~1985

- Scientific tool

CAx Sunrise



~2000

- Computer-Aided Tools in R&D

Virtual Prototyping

~2015

- Collaborative Model-based Engineering

Digital Twin Era

~2022

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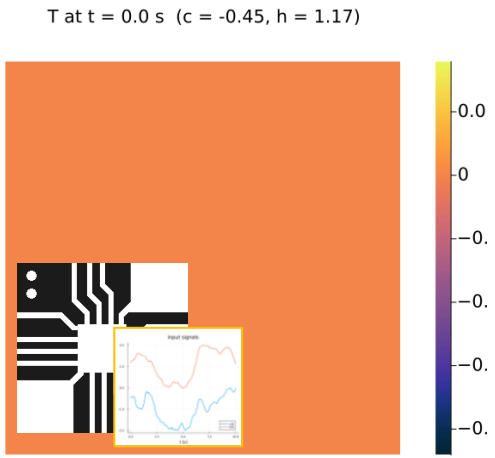
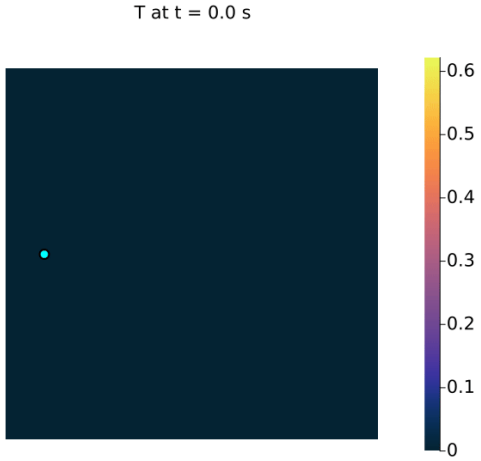
Real-Time Algorithms for Digital Twins
Exercise Examples



"Electronics Cooling"

$$\partial_t T = \Delta T + c(t)q(x)$$

- Linear PDE
- Transient cooling and heating
- Geometry via pictures
- The poster-child for ROM

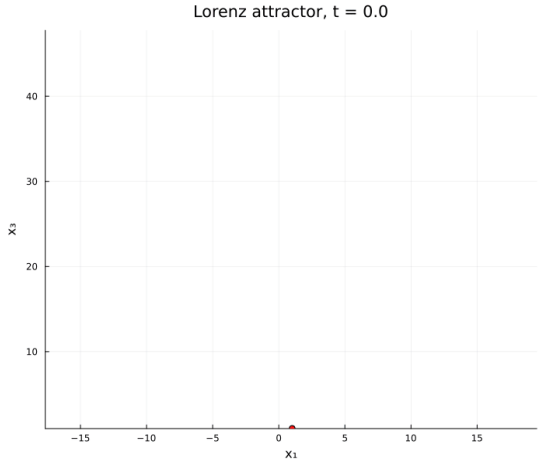


"Selective Laster Melting"

$$\partial_t T = \Delta T + q(x,t)$$

- Linear PDE
- Heating via a moving heat source
- The nightmare for ROM

Real-Time Algorithms for Digital Twins
Exercise Examples



Lorenz System

$$\partial_t x = \sigma(y - x)$$
$$\partial_t y = x(\rho - z) - y$$
$$\partial_t z = xy - \beta z$$

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